One Loop Corrections to the W mass

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Overview

1. Basics
2. Renormalisation
3. Making predictions
4. Predicting the 1 loop correction to the W mass

Basics

- Renormalise theory to one-loop (on-shell)
- With a set of tree-level relations and input parameters, decide what should be computed in terms of inserted quantities.
- Here, find the one-loop correction to the muon decay $\rightarrow \Delta r$
- Δr contains various contributions from QED + EW
- Express corrections to m_W in terms of pieces of Δr
- For reference, $\rho \equiv \frac{m_W^2}{c_W^2 m_Z^2}$

Renormalisation setup

The highlights:

- Must renormalise RH, LH fields separately.
- $V_\mu^a = (Z_2^V)^{1/2} V_{\mu\,R}^a \qquad$ where $V = B, W$
- $g_i = Z_1^{V_i} (Z_2^{V_i})^{-3/2} g_{i\,R}$
- Appropriate renormalisation leads to tadpoles not participating in renormalised quantities.
- $\begin{pmatrix} \delta Z_i^\gamma \\ \delta Z_i^Z \end{pmatrix} = \begin{pmatrix} s_W^2 & c_W^2 \\ c_W^2 & s_W^2 \end{pmatrix} \begin{pmatrix} \delta Z_i^W \\ \delta Z_i^B \end{pmatrix}$

ex. contributions to W self energy

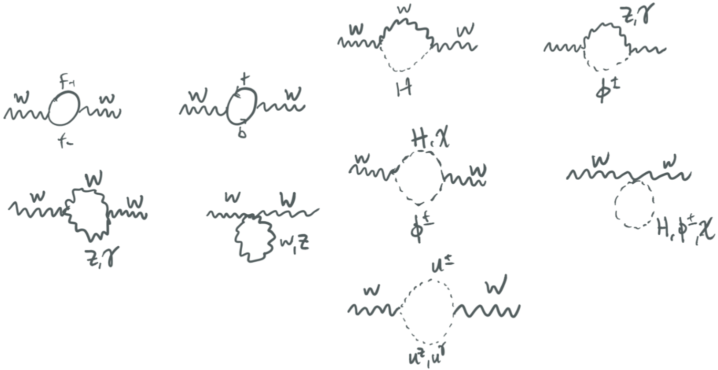


Figure: 1 loop contributions to the W self energy, including unphysical states

Coupling constant renormalisation

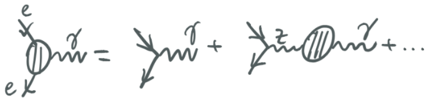


Figure: $Z - \gamma$ mixing contributing to the QED vertex

- On-shell scheme says in the limit of vanishing momenta, recover Thomson limit for QED
- Need expressions for renormalisation of electric charge
- $e^2 = \frac{g_1^2 g_2^2}{g_1^2+g_2^2} = \left[\frac{g_1^2 g_2^2}{g_1^2+g_2^2}\right]_R (1 + 2\delta Z_1^\gamma - 3\delta Z_2^\gamma) \equiv e_R^2 \left(1 + 2\frac{\delta e}{e}\right)$
- $\frac{\delta e}{e} = \frac{1}{2}\Pi^\gamma(0) - \frac{s_W}{c_W}\frac{\Sigma^{\gamma Z}(0)}{m_Z^2}$

Weak mixing angle

- Also note that one can renormalise the weak mixing angle
- This is a difficult object to assign physical meaning to across schemes
- First define, $s_W^2 = 1 - \frac{m_W^2}{m_Z^2}$
- $s_W^2 = 1 - \frac{m_W^2}{m_Z^2} + \frac{m_W^2}{m_Z^2}\left(\frac{\delta m_Z^2}{m_Z^2} - \frac{\delta m_W^2}{m_W^2}\right)$

Input parameters

- QED: m_f, α
- EW: add m_W, m_Z, m_H
- Can use tree level relations to exchange input parameters
- Relevance and measurement dictates a trade of m_W for G_μ
- So: $\alpha, G_\mu, m_f, m_Z, m_H$
- Other sets of parameters useful for different cases (experiment)

Muon decay constant

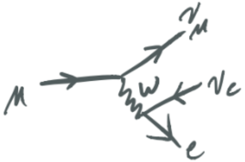


Figure: tree-level muon decay

- Account for all one loop corrections
- QED corrections contribute $\rightarrow$ contribute to prediction for W mass.
- At tree level, $G_\mu = \frac{\sqrt{2}e^2}{8s_W^2 m_W^2}(1 + \frac{\Sigma^{WW}(0)}{m_W^2} + \delta_{V+B}) \equiv \frac{\sqrt{2}e^2}{8s_W^2 m_W^2}(1 + \Delta r)$
- V+B are vertex & box corrections.

Tree-level predicion

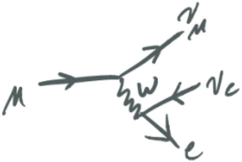
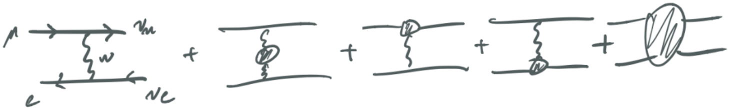
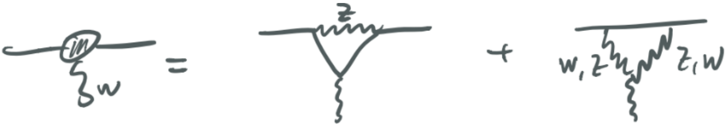


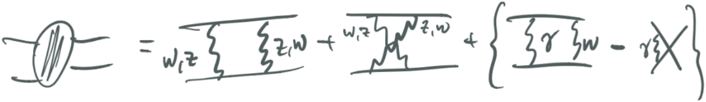
Figure: tree-level muon decay

- Measure the muon lifetime, s_W^2, Z mass, α.
- Using the tree-level relation, $m_W \sim 77.5$ GeV

Contributing EW diagrams to δ_{V+B}

- 

- 

- 

Δr

- Following the renormalisation procedure gives:

- $$\Delta r = \frac{\sqrt{2}e^2}{8s_W^2 m_W^2}\Bigg\{\Pi^\gamma(0) - \frac{c_W^2}{s_W^2}\left(\frac{\delta m_Z^2}{m_Z^2} - \frac{\delta m_W^2}{m_W^2}\right) + \frac{\Sigma^{WW}(0) - \delta m_W^2}{m_W^2} + 2\frac{c_W}{s_W}\frac{\Sigma^{\gamma Z}(0)}{m_Z^2} + \frac{\alpha}{4\pi s_W^2}\left[6 + \frac{7-4s_W^2}{2s_W^2}\log c_W^2\right]\Bigg\}$$

- Partition Δr into separate contributions

- $\Delta r = \Delta\alpha - \frac{c_W^2}{s_W^2}\Delta\rho + \Delta r_{rem}$

$\Delta\alpha$

- Photon polarisation part of $\frac{\delta e}{e}$ contributes
- Split into three parts: leptons, heavy quarks, light quarks
- Top is heavy, light is problematic
- $\Pi_t^\gamma(m_Z^2) \approx \frac{\alpha}{\pi} Q_t^2 \frac{m_Z^2}{5m_t^2} \quad \to \text{decoupling}$
- $\mathrm{Re}\ \Pi_l^\gamma(m_Z^2) = \frac{\alpha}{3\pi}\left(\frac{5}{3} - \log\frac{m_Z^2}{m_l^2}\right)$
- $\Delta\alpha^{(5)}_{light} \to \Delta\alpha_{hadrons}$

Hadronic contribution to $\Delta\alpha$

- Inputing quark masses difficult.
- Use optical theorem to express contribution in terms of measured cross-sections
- Measure $R(s) = \frac{\sigma(e^+e^- \to \gamma^* \to \mathrm{hadrons})}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)}$
- $\Delta\alpha_{hadrons}(m_Z^2) = -\frac{\alpha m_Z^2}{3\pi} \mathrm{Re} \int_{4m_\pi^2}^{\infty} ds \frac{R(s)}{s(s - m_Z^2 - i\varepsilon)}$
- After calculating this, one could work backward and see what quark masses to input & use perturbative QCD

$\Delta\rho$

- $\Delta\rho = \frac{\Sigma^{ZZ}(0)}{m_Z^2} - \frac{\Sigma^{WW}(0)}{m_W^2}$
- NCs don't mix components of doublets, CCs do. i.e. $\Delta\rho$ sensitive to doublet mass splitting.
- $\Delta\rho_f = N_C \frac{\alpha}{16\pi s_W^2 c_W^2 m_Z^2} \left(m_1^2 + m_2^2 - \frac{2 m_1^2 m_2^2}{m_1^2 - m_2^2} \log \frac{m_1^2}{m_2^2} \right)$
- Simple for the third gen.: $\Delta\rho_{top} = N_C \frac{\alpha}{16\pi s_W^2 c_W^2} \frac{m_t^2}{m_Z^2}$
- Related to correcting s_W^2

Scalar contribution to $\Delta\rho$

- Diagrams involving unphysical states must be included
- $\Delta\rho_H$ is a pretty heinous calculation result involving combinations of m_H, m_Z, m_W.
- For a heavy higgs ($m_H \gg m_W$), $\Delta\rho_H \simeq \frac{g_2^2}{16\pi^2}\frac{3s_W^2}{4c_W^2}\log\frac{m_H^2}{m_W^2}$

$\Delta\rho$ with regards to custodial symmetry

- $\Delta\rho_H \propto s_W^2$ or $\propto g_1^2 \equiv g'^2$
- In the limit of $s_W \to 0, \rho_H \to 0$
- Expected from custodial $SU(2)_{L+R}$ symmetry, equivalent to $g' \to 0$
- Then ρ_H a measure of the $SU(2)_{L+R}$ breaking by Y

Δr_{rem}

- Typically smaller than the other contributions by several times
- Light fermions $\Delta\rho_{rem} \simeq \frac{\alpha}{4\pi s_W^2}(1 - c_W^2/s_W^2)\frac{N_{Cf}}{6}\log c_W^2(1 + \delta_{QCD})$
- $\delta_{QCD} \sim 4\%$ at the Z pole
- contribution from higgs

Predicting the W mass

- $\Delta\rho_{rem}$ has negligible contribution from Higgs
- $\Delta\rho_{rem}^{top} \simeq -0.001$
- $\Delta\alpha_{hadrons}(m_Z^2) \simeq 0.028$, ignore small contribution from top
- $\Delta\alpha_f(m_Z^2) \simeq 0.0595$
- $\Delta\rho_H \simeq (s_W^2/c_W^2)\,0.0017$
- $\Delta\rho_t \simeq (s_W^2/c_W^2)\,0.032$
- This gives $m_W \simeq 79.1$ GeV

Resumming

- Separate treatments of resumming large terms in $\Delta\alpha, \Delta\rho$
- QED: $1 + \Delta\alpha \to \frac{1}{1-\Delta\alpha}$
- For a heavy top, need to include higher order results
- $\Delta\overline{\rho} = N_C \frac{G_\mu m_t^2}{8\pi^2\sqrt{2}} \left(1 + \rho^{(2)} \frac{G_\mu m_t^2}{8\pi^2\sqrt{2}}\right)$ where $\rho^{(2)} = 19 - 2\pi^2$ $(m_H \ll m_t)$
- Then $(1 + \Delta r) \to \frac{1}{1-\Delta\alpha} \frac{1}{1+c_W^2 \Delta\overline{\rho}/s_W^2} + \Delta r_{rem}$
- With this, $m_W^{pred} \simeq 80.3$ GeV

References

- Jegerlehner, F. (1990). Renormalizing the standard model. Testing the Standard Model, 476-590.
- Hollik, W. (1995). Renormalization of the standard model. In Precision tests of the standard electroweak model (pp. 37-116).